# UK Junior Mathematical Olympiad 2002

Organised by The United Kingdom Mathematics Trust

Tuesday 11th June 2002

## RULES AND GUIDELINES :
## READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators *and measuring instruments* is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required.* Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

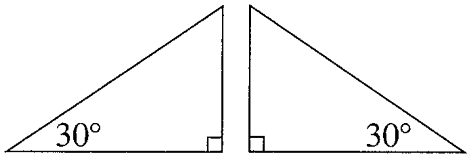
   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   ***Do not hand in rough work.***
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first hour so as to allow at least one hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Numerical answers must be FULLY SIMPLIFIED, and EXACT using symbols like $\pi$, fractions, or square roots if appropriate, but NOT decimal approximations.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!**

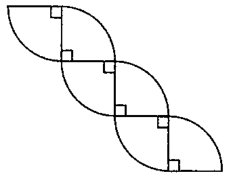
## Section A

**A1** Two identical right-angled triangles are made out of cardboard. How many different shapes can be produced by gluing the two triangles together, matching two equal edges?

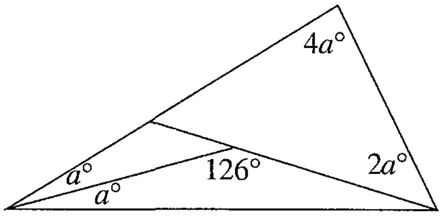



**A2** Calculate $\left[(-1) + (-1)^2\right] \div \left[(-1) - (-1)^2\right]$.

**A3** In the figure shown, each arc is of radius $r$. What is the perimeter of the figure?



**A4** What is the value of $a$ in the triangle shown?

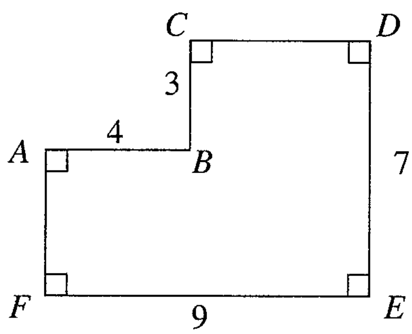



**A5** The sum of seven consecutive even numbers is 2002. What is the smallest of these seven numbers?

**A6** Find two whole numbers, neither of which ends in the digit zero, whose product is ten thousand.

**A7** Two thirds of five sixths of a number $X$ is the same as three quarters of four fifths of a number $Y$. What is the value of $\frac{X}{Y}$ as a fraction in lowest terms?

**A8** Calculate the length of $BE$ in the figure shown.




**A9** Given that $\frac{1}{x + 6} = 4$, find the value of $\frac{1}{x + 8}$.

**A10** Two boxes, P and Q, each contain 3 jewels. When a jewel worth £5000 is transferred from P to Q, the average value of the jewels in each box increases by £1000. What is the total value of all 6 jewels?

## Section B

Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

**B1** A number like 4679 is called an *ascending* number because each digit in the number is larger than the preceding one.

(i) How many ascending numbers are there between 1000 and 2000?

(ii) How many ascending numbers are there between 1000 and 10000?

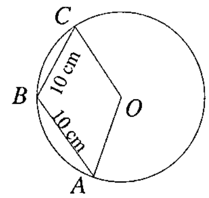
**B2** Five teams played in a competition and every team played once against each of the other four teams. Each team received three points for a match it won, one point for a match it drew and no points for a match it lost. At the end of the competition the points were:

Yellows 10, Reds 9, Greens 4, Blues 3 and Pinks 1.

(i) How many of the matches resulted in a draw?

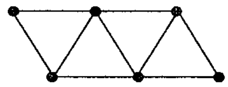
(ii) What were the results of Greens' matches against the other four teams?

**B3** In the diagram, $O$ is the centre of the circle. The lengths of $AB$ and $BC$ are both 10 cm. The area of quadrilateral $OABC$ is 120 cm$^2$. Calculate the radius of the circle.



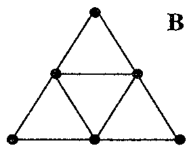

**B4** (i) Network A has nine edges which meet at six nodes. The numbers 1, 2, 3, 4, 5, 6 are placed at the nodes, with a different number at each node. Is it possible to do this so that the sum of the two numbers at the ends of an edge is different for each edge?

Either show a way of doing this, or prove that it is impossible.



**A**

(ii) Repeat the same procedure for Network B, i.e. show that it is possible to place the six numbers so that the sum of the two numbers at the ends of an edge is different for each edge, or prove that it is impossible to do so.



**B**

**B5** *ABCDE* is a pentagon in which triangles *ABC*, *AED* and *CAD* are all isosceles, $AC = AD$, $\angle CAD$ is acute. Interior angles *ABC* and *AED* are both right angles.

Draw a sketch of pentagon *ABCDE*, marking all the equal sides and equal angles. Show how to fit four such identical pentagons together to form a hexagon. Explain how you know the pentagons will fit exactly.

**B6** A game for two players uses four counters on a board which consists of a $20 \times 1$ rectangle. The two players take it in turns to move one counter. A turn consists of moving any one of the four counters any number of squares to the right, but the counter may not land on top of, or move past, any of the other counters. For instance, in the position shown, the next player could move $D$ one, two or three squares to the right, or move $C$ one or two squares to the right and so on.


The winner of the game is the player who makes the last legal move. (After this move the counters will occupy the four squares on the extreme right of the board and no further legal moves will be possible.)

In the position shown above, it is your turn. Which move should you make and what should be your strategy in subsequent moves to ensure that you will win the game?